NUMERICAL MODELING OF AN OPTIMAL HEAT ACCUMULATOR WITH PHASE CHANGE

O. V. Dikhtievskii, G. V. Konyukhov, O. G. Martynenko, and I. F. Yurevich

UDC 536.42:621.31.61

The paper provides a procedure for and the results of the mathematical modeling of an optimum heat accumulator with phase change which operates with periodic recharging from a solar energy concentrator. Calculations have been carried out for two types of heat transfer media with laminar and turbulent flow conditions of the media.

Heat accumulators (HAs) are included in energy systems which have variations in the capacity of the heat source or in the energy requirements and are used for purposes of stabilization and increasing the efficiency and energy economy of such systems. Heat accumulators are based on heat-capacity properties, phase changes, reversible chemical reactions, or adsorption–desorption processes of a material, and are classified according to the level of the working temperature, the scale of the application, and the length of the charging–discharging cycle [1-3].

The HA being considered in this paper is a high-temperature, short-term device based on a phase-change phenomenon (melting–solidification) which is charged from a solar energy concentrator and operates as part of a transport energy unit. As the initial construction of the device (the choice of which can considerably influence the results of the investigation) a HA design was selected (Fig. 1) whose operating principle has been given in the description of the device [4]. The main difference of the HA being investigated from analogous devices [2] consists of the method of inputting the radiant stream, which falls onto a receiving window filled with focons which undergo a continuous transition into hollow narrow light channels with partial absorption, and of a dense (bulk) packing of heat-accumulating elements (HAEs). This construction of the radiant energy receiver makes it possible to put together a HA package made up of standard modules each of small capacity, and also makes it possible to achieve a significant decrease in the volume of the HA, its mass, and the area of its shadow on the concentrator compared to the corresponding parameters for analogous devices [2]. The latter consist of conico-cylindrical volumes with the spaces on the internal side surfaces and base filled with HAEs, which leaves a considerable part of the volume empty with a concentration of the mass in the side walls near the surface layer.

For developing an optimal HA within the framework of the design which has been selected [4], i.e., in order to determine its optimum characteristics [with respect to mass, hydraulic,

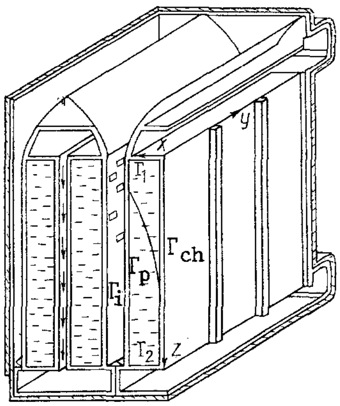

Fig. 1. Construction of heat accumulator.

Academic Science Complex, A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 61, No. 5, pp. 749-755, November, 1991. Original article submitted January 17, 1991.

0022-0841/91/6105-1345$12.50

geometrical and light-absorbing properties (in the light channels)] and to analyze the system, it is necessary to carry out mathematical modeling of the device. Its design is represented in the form of a sequence of interacting computational blocks:

1. The selection (or correction) of the initial constructional model of the HA.

2. The specification in the initial approximation of the field of the temperature T^* at the surface of the heat removal channels at the moment $\tau = \tau^*$ at which the illuminated period of the operating cycle of the HA begins.

3. Specification of the optimization conditions and of the free parameters.

4. Solution of the problem of optimizing the HA. The results of this give the mass, hydraulic, and geometric characteristics of the optimum device.

5. Solution of the problem of heat transfer in the HAEs for the parameters resulting from block 4 for the shadowed period (discharge of the HA) of the operating cycle and up to $\tau = \tau^*$. The results of this provide the thermal state of the HAEs in this period, and in particular, the temperature field T_c^* (τ^*, P) of the heat removal surfaces. Calculations are made of the thermoelastic state of the uniformly loaded walls of the channels with subsequent checking of the conditions for strength, and corrections are made, as required, for the wall thickness δ and cross-sectional dimensions of the channel. The conditions are passed back to block 4 through the calculated distribution $T^* = T_c^*$. The iteration process between block 4 and block 5 is continued until the geometrical parameters of the HA and the fields of T^* and T_c^* converge to the required accuracy.

6. Specification (or correction) of the distribution of the heat flux density q over the surface of the light channel.

7. Solution of the problem of heat transfer in the HAEs for the parameters resulting from blocks 4-5 and taking into account the data of block 6 for the illuminated period of the operating cycle (charging of the HA). The results give the thermal state of the HAEs after the period of calculation, and in particular, the temperature field T_c^* (τ_c^*, P) at the moment of time τ_c^* corresponding to the minimum temperature of the heat transfer medium at the exit from the HAEs. This is fed back to block 6. The iteration process between block 6 and block 7 is continued until no further correction to the flux distribution q is required to ensure stability of the characteristics of the heat transfer medium at the exit of the HAEs and uniformity of the melting of the heat accumulating material (HAM) over its volume. In the last iteration calculations are also made of the thermoelastic stresses in the walls of the heat-receiving channels, with corrections, as required, in the wall thicknesses or in the cross-sectional dimensions of the channel, and then the information is transferred to block 4 with the specification of new calculated values $\tau^* = \tau_c^*$ and $T^* = T_c^*$.

In order to establish that stability exists in the operation of the HA, the calculations are repeated for several working cycles. Still further investigations can be carried out with corrections either to the conditions of optimization and the free parameters (passing back to block 3), or to the initial constructional design (passing back to block 1).

The target function of the optimization problem is the mass m of the HA, which is made up from the mass of the heat insulation, the focons, the devices for feeding and drawing off the heat transfer medium, the heat accumulating material, and the walls of the HAEs. The mass is a function of the geometric dimensions of the HA, its capacity, the density of the heat accumulating material, the density of the material of construction, the angle of convergence of the focons, the latent heat of phase change, and the type of heat insulation:

$$m = g_0 (L_{ch}, \rho_{HAM}, \rho_w, \delta, \chi, R_h, a, b, \gamma, E, r). \quad (1)$$

In order to ensure the temperature driving force required in the energy system, a lower limit is introduced for the temperature of the heat transfer medium at the outlet from the HAEs:

$$(T_p - \Theta_{ch})/(T_p - \Theta_0) \leqslant \varepsilon_T. \quad (2)$$

Maintaining a high efficiency of the system requires the imposition of an upper limit on the relative pressure drop of the heat transfer medium in the channels of the HA:

$$(P_0 - P_{ch})/P_0 \leqslant \varepsilon_P. \quad (3)$$

A limit is also placed on the leakage of heat through the thermal insulation of the HA:

$$q_{out}\tau_c / E \leqslant \varepsilon_F \tag{4}$$

and the losses in reflecting the radiant flux directed from the concentrator onto the focons of the receiving window of the HA:

$$(2n_L + 1)\gamma + \varphi \leqslant \frac{\pi}{2} . \tag{5}$$

By using analytical solutions of the steady-state equations of heat transfer and of the motion of the heat transfer medium averaged over the cross section of the heat removal channels, with the condition that the temperature at the surface Γ_{ch} of the channels is represented in the form of the power series

$$T|_{\Gamma_{ch}} = \sum_{i=1}^{l} A_i z^i, \tag{6}$$

and taking into account the equation of state $P = \rho R_T \Theta$, the expression for the mass flowrate $Q = \rho uF$, and semi-empirical formulas for the coefficient of hydraulic resistance ξ and the heat transfer coefficient α, the mass function (1) and the optimization conditions (2)-(5) can be represented in the form of sums of polynomials of nonnegative variables. As a result, the mathematical statement is reduced to a problem in geometric mathematical programming [5]:

$$m = g_0(\bar{x}) \to \min, \ g_i(\bar{x}) \leqslant 1, \ i = 1, 2,$$

where

$$g_i(\bar{x}) = \sum_{k=1}^{n_i} c_i^k \prod_{l=1}^{m} x_l^{a_{k,l}}, \ i = 0, 1, 2; \ c_i^k = \text{const}, \ \bar{x} \in R^m.$$

The solution of the latter is carried out by the method of condensation using a polynomial approximation algorithm [6]; the programming was carried out in FORTRAN-4 language.

The determination of the thermal state of the HAEs, a diagram of which is given in Fig. 1, consisted of solving the problem of a two-dimensional unsteady-state heat transfer in the zone of the HAM with a moving phase boundary and the one-dimensional quasi-steady-state problem in the heat recovery channel. The simplified statement of the problem for the heat transfer medium is valid, since the characteristic time for heat transfer in the channel is much smaller than the analogous parameter for the zone of the HAM, and the channel length is much greater than its width.

In solving the Stefan problem use is made of the principle of "spreading out" the heat of phase change with respect to the heat capacity of the HAM in the Δ-vicinity of the temperature of phase change; in this case, the boundary of the front cannot be clearly isolated [7]. By introducing the Dirac δ-function, the heat conduction equations for the solid and liquid phases together with the conditions at the phase interface Γ_p can be represented in the form

$$[c_1(T) + r_1\delta(T - T_p)]\frac{\partial T}{\partial \tau} = \operatorname{div}(\lambda(T)\operatorname{grad} T), \tag{7}$$

where $c_1(T) = \rho(T)c_P(T)$; $r_1 = \rho(T)r$;

$$\rho(T) = \begin{cases}\rho_S, & T < T_p,\\ \rho_L, & T > T_p,\end{cases} \quad c_P(T) = \begin{cases}c_{PS}, & T < T_p,\\ c_{PL}, & T > T_p,\end{cases} \quad \lambda(T) = \begin{cases}\lambda_S, & T < T_p,\\ \lambda_L, & T > T_p.\end{cases}$$

The boundary conditions are:

on the ends of the heat-accumulating elements,

$$\left.\frac{\partial T}{\partial z}\right|_{\Gamma_1} = \left.\frac{\partial T}{\partial z}\right|_{\Gamma_2} = 0; \tag{8}$$

on the surfaces of the heat removal channels,

$$\lambda (T) \frac{\partial T}{\partial x}\Big|_{\Gamma_{ch}} = \alpha (T|_{\Gamma_{ch}} - \Theta (z, \tau)), \tag{9}$$

where the temperature of the heat transfer medium $\Theta(z, \tau)$ is recalculated on the basis of the energy balance equation

$$Qc_P \frac{\partial \Theta}{\partial z} = \alpha \chi (T|_{\Gamma_{ch}} - \Theta), \ \Theta (0) = \Theta_0,$$

for each time step of the difference analog (7);

on the input surfaces,

$$\lambda (T) \frac{\partial T}{\partial x}\Big|_{\Gamma_i} = q (z, \tau). \tag{10}$$

For finding numerical solutions of Eq. (10) with the boundary conditions (8)-(10) given above, the δ-function is replaced by the delta-like function $\delta(T - T_p, \Delta)$, which is similar to it. In this case, the temperature of the melt must exceed the temperature of phase change T_p by the amount Δ.

The initial condition then has the form

$$T (\tau = 0) = T_p + \Delta.$$

The expression $c_1(T) + r_1\delta(T - T_p, \Delta)$ from Eq. (7) is approximated by the function

$$\rho (T) \{A \exp (- [(T - T_p)/\Delta]^2) + B \operatorname{th} (3 (T - T_p)/\Delta) + C\};$$

the coefficients A, B, and C are determined as functions of the characteristics of the HAM from the conditions

$$c_P (T_\ell) = c_{PS}, \ c_P (T_u) = c_{PL}, \ \int_{T_\ell}^{T_u} c_P (T)\, dT = c_{PS} (T_p - T_\ell) + c_{PL} (T_u - T_p) + r,$$

where T_ℓ and T_u are the lower and upper boundaries of the range of working temperatures of the HAM in the problem being considered.

The Stefan problem stated in this way is solved by the finite element method. On the basis of the temperature fields which are obtained, the same method is used to calculate the thermoelastic stresses in the walls of the heat extraction channels of the HAEs and then the strength limits are checked:

$$|\sigma_w| \leqslant \sigma_\ell/2.$$

The temperature values on the surfaces of the heat recovery channels at the moment of time $\tau = \tau^*$ are approximated by the power polynomial (6), and are used for the next iterative solution of the optimization problem. The calculations of the thermostressed states are carried out to the thin-plate approximation taking into account the transverse loading P_0 [8].

The mathematical modeling of a HA of the assumed construction [4] was carried out for a series of variants of the device, and in particular, with turbulent and laminar flow conditions of the heat transfer medium in the heat recovery channels, with different heat-accumulating materials and heat transfer media, and with variations of the other free parameters in the optimization problem.

The determination of the geometric, mass, and hydraulic characteristics of the optimum HA according to the procedure proposed above under conditions of turbulent flow of the heat transfer medium in the channels requires three to four iterative solutions of the optimization and Stefan problems, and two iterations under laminar flow conditions. In the latter case the calculations are simplified, since their results depend only slightly on the limit (2) on the heating of the heat transfer medium in the channels (in contrast to the case with turbulent flow), and consequently, they depend only slightly on the solution of the Stefan problem after the shadowed period.

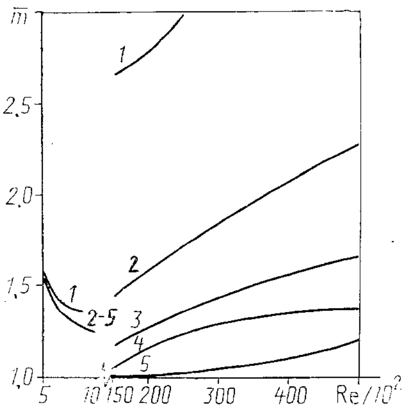

Fig. 2

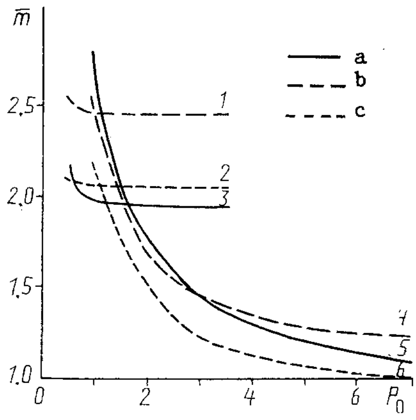

Fig. 3

Fig. 2. Dependence of the mass of the optimal HA on the Reynolds number for various pressures P_0 in atm: 1) 2; 2) 4; 3) 6; 4) 8; 5) 10.

Fig. 3. The mass of the optimal HA as a function of the pressure P_0 (atm): 1-3) under laminar flow conditions; 4-6) under turbulent flow conditions; a) LiF; b) LiOH; c) 80% LiOH—20% LiF.

The characteristics of the optimal HA differ appreciably for these two cases. For the case of trubulent flow conditions in the channels, the longitudinal dimension of the apparatus L_z exceeds the transverse dimension L_{xy} by 2 to 38 times over the range of capacities 2.5-25 kW and pressures P_0 = 1-10 atm. In the laminar construction, a flat structure results, and the ratio L_z/L_{xy} falls from 0.16 to 0.08. From an analysis of the mass characteristics of HAs of equal capacity it follows that equipment having laminar flow conditions of the heat transfer medium has a mass 20-30% greater than that of the HA variants having turbulent flow conditions, differs by being more complex to manufacture (a considerably larger number of HAEs is required, and these have narrow, high-precision channel sizes), and there are considerable heat losses by re-radiation from the receiving window during the charging period (10-15% compared to 1-3% for the equipment with turbulent flow conditions). Figure 2 shows the masses of HAs of capacity 25 kW for both flow regimes as a function of the Reynolds number.

The choice of the HAM in the range of working temperatures 700-1100 K is determined by the requirements with respect to cyclability, the value of the heat of phase change, and the chemical stability of the materials of construction to it [1]. Investigations of HAs carried out for lithium fluoride LiF, lithium hydroxide LiOH, and the eutectic 80% LiOH—20% LiF show that the minimum mass characteristics under turbulent flow conditions in the channels occur in a unit with the eutectic as the HAM, while under laminar flow conditions, this occurs with lithium fluoride.

The results given below were obtained for the case of turbulent flow conditions of the heat transfer medium in the heat recovery channels.

Calculations for equipment using xenon and helium—xenon mixtures as the heat transfer medium show that the second variant has an advantage as the result of the greater heat transfer intensity. For example, for a capacity of 25 kW the length of the heat recovery channels appears to be 40-45% shorter than in the case where xenon is used as the heat transfer medium, and the mass of the HA is smaller by 30-45% over the range of pressures at the inlet to the heat recovery channels from 4 to 10 atm.

As the pressure P_0 is increased the mass of the HA falls off asymptotically, reaching a practically constant value m* at the pressure P_0* (see Fig. 3). The value of P_0* is a function of the capacity of the unit, the type of heat transfer medium, and the HAM, and the limiting value of mass m* of the HA is reached at smaller values of the pressure P* as the capacity of the unit increases.

The ratio of the heat recovery channel lengths of the optimum HA calculated by numerical modeling and of the ideal HA characterized by a constant value equal to T_p of the temperature of the heat recovery surfaces has values of 1.3-3.6 for the different variants of the equipment and as a function of condition (2). When the condition is specified that the temperature

Θ_c of the heat transfer medium at the outlets of the channels should be close to T_p, then about a half of the outlet part of the HAEs remains filled with melt at the end of the shadowed period of operation.

The final modeling results show that by means of varying the distribution of the heat flux being absorbed by the surfaces of the light-pipes it is not possible to achieve a state of complete melting of the HAM during the illuminated period: 6-7% of it in the corner zones of the HAEs remains in the solid form, while a part of the melt appears to be overheated. At the same time, the distributions which have been selected make it possible to satisfy the stated optimization conditions over specified ranges.

The analysis of the solution of the problem of thermoelasticity in the plates taking into account the transverse loading P_0 indicates the necessity of modifying the slot channels for heat removal (for circular channels the strength conditions are more than satisfied), for example, by introducing transverse separating walls. In this case at a pressure of P_0 = 10 atm, for example, the ratio of the size of the transverse sides of the channels in the cross section should not be greater than four. From the analysis of the thermal stresses it was also concluded that it was preferable to use the eutectic mixture as the HAM both because of the smaller mass of the HA in this case and also from the point of view of the strength of the construction. In this variant of the HA less massive characteristics of the device are found at the lower values of the pressure P_0 of the heat transfer medium, and hence, at smaller stresses in the construction elements.

As a result of the investigations of the HA arrangement being considered here [4] optimal parameters have been obtained for several variants of the equipment; the results confirm that the efficiencies exceed those of analogous foreign HAs. For example, approximate calculations for the analog [2] compared with the data obtained for the construction being investigated here show that the energy-specific mass of the analog exceeds the corresponding parameter for this construction by 1.3-2.0 times in the capacity range from 2.5 to 25 kW.

NOTATION

m) mass; E) quantity of energy; q) heat flux density; T, Θ) temperatures of HAM [heat accumulating material] and of heat transfer medium; P) pressure; c_p, λ, α) heat capacity, thermal conductivity, and heat transfer coefficient; r) latent heat of phase change; R_T) gas constant of heat transfer medium; Q) mass flow rate; ρ) density; u) velocity; F, χ) area and perimeter of transverse cross section of channel; R_h) hydraulic radius; L) linear dimension; a) thickness of layer of HAM in symmetrical part of HAE [heat accumulating element]; c) thickness of light pipe; δ) wall thickness; γ, φ) angles between the plane of symmetry of the focon and, respectively, the surface of the focon or the direction of the incoming radiant flux; n_L) maximum number of reflections of the light beam from the surfaces of the focon; ε_P, ε_T, ε_E) relative deviations of P, T, and E; τ) time; σ) stress; σ_ℓ) ultimate [limiting] strength; subscripts: c) calculated; ch) channel (or final value); L) liquid phase; 0) initial value; p) phase change; s) solid phase; w) at channel wall.

LITERATURE CITED

1. O. V. Dikhtievskii, I. F. Yurevich, and O. G. Martynenko, Heat Accumulators [in Russian], Preprint No. 27, Inst. of Heat and Mass Transfer, Acad. of Sciences of the BSSR, Minsk (1989).
2. N. M. Faget, W. M. Fraser, Jr., and W. E. Simon, in: Proc. 20th Intersoc. Energy Convers. Engng. Conf., Miami Beach, Fl., August, 1985, Vol. 2, Pennsylvania (1985).
3. Accumulation of Energy and Means for Improving the Operating Efficiencies of Power Stations and for Improving Energy Economies [in Russian], Proc. of All-Union Scientific and Technical Conference (Moscow, October, 1985), Vols. 1-2, Moscow (1986).
4. Positive Solution VNIIGPE, No. 4693193/06 of November 17, 1989.
5. D. Wilde, Optimum Design [Russian translation], Moscow (1981).
6. A. V. Pokataev, Computer Software [in Russian], No. 55, Minsk (1985), pp. 144-152.
7. B. M. Budak, E. N. Solov'eva, and A. B. Uspenskii, Zh. Vychis. Mat. Mat. Fiz., 5, 827-840 (1965).
8. B. Boli and J. Weiner, Theory of Temperature Stresses [Russian translation], Moscow (1964).